Spatial skills as a predictor of first grade girls' use of higher level arithmetic strategies

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A R T I C L E   I N F O
Article history:
Received 9 September 2011
Received in revised form 2 August 2012
Accepted 6 August 2012

Keywords:
Spatial skills
Verbal skills
Arithmetic
Gender
Elementary school
Strategies

A B S T R A C T

Girls are more likely than boys to use counting strategies rather than higher-level mental strategies to solve arithmetic problems. Prior research suggests that dependence on counting strategies may have negative implications for girls' later math achievement. We investigated the relation between first-grade girls' verbal and spatial skills and the strategies they used to solve arithmetic problems. The present findings are consistent with our hypothesis that individual differences in girls' use of higher-level mental strategies are related to differences in their spatial abilities. Spatial skills positively predicted frequency of use of both higher-level mental strategies (retrieval and decomposition), while verbal skills only contributed to the use of decomposition. Furthermore, the rate of use of the least sophisticated counting strategy was negatively related to spatial skills.

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In mathematics, basic skills are necessary for advanced thinking and problem solving. The ability to efficiently and accurately perform basic arithmetic operations is one such fundamental skill. Multiple strategies (e.g., counting, retrieval) can be used to solve arithmetic problems, some of which are more efficient and sophisticated than others. As early as kindergarten, there are individual differences in the frequency with which children use different strategies. These early differences predict later differences in mathematics performance, particularly for girls (Carr, Hettinger Steiner, Kyser, & Biddlecomb, 2008; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Thus, it is important to understand some of the cognitive factors that contribute to individual differences in early strategy use. In the present study, we investigated whether young girls' spatial and/or verbal abilities contribute to their use of higher-level, more sophisticated strategies.

1. Arithmetic strategies

Arithmetic problems can be solved using a variety of strategies. Generally, children use one of four strategies to solve arithmetic problems: count-all, count-on, decomposition, and retrieval. The count-all strategy involves counting out each addend and then counting the total (e.g., to solve 5+3, a child would first count to 5, then count to 3, then finally count from 1 to 8). The count-on strategy involves counting up from one addend, the value of the second addend (e.g., to solve 5+3, a child would count 6, 7, 8). Decomposition involves decomposing a problem into simpler problems; for example, to solve 5+6, a child might first add 5+5 to get 10 and then add one more to arrive at 11. It often involves multiple steps, including remembering to add or subtract back numbers that were added or taken away from the original addends. The last strategy, retrieval, involves recalling the solution to an arithmetic problem from memory.

At any given time, children, and even adults, use a mix of these strategies to solve arithmetic problems. For example, on smaller easier problems, children are more likely to use retrieval, while on larger more difficult problems children are more likely to use count-all or count-on (Ashcraft, 1982; Siegler & Jenkins, 1989). Thus, in the same session, an individual might use count-on to solve one problem, retrieve the answer from memory to answer the next problem, and use decomposition to solve another problem. In addition, there are individual differences within children of the same age in terms of the frequency with which they use different strategies. For instance, if two first graders are asked to solve 10 arithmetic problems, one child might solve all 10 of them by using a count-all strategy, whereas the other might solve 5 of the 10 problems using a count-on strategy and the other 5 by retrieving the answer from memory.

Individual differences in the frequency with which young children use different strategies are important because strategy usage is related to later math achievement (e.g., Carr & Alexeev, 2011). Mental strategies,
such as decomposition and retrieval, are generally considered higher-level strategies than count-all and count-on for a number of reasons. First, relative to counting strategies, decomposition and retrieval are more efficient; problems solved using decomposition and retrieval are solved more quickly than ones using counting strategies (e.g., Ashcraft & Flanagan, 1982). Second, decomposition and retrieval draw on memory based mental procedures that depend on prior knowledge of number facts (Ashcraft & Stazyk, 1981; Geary, 2011), and tend to emerge only after substantial practice solving arithmetic problems (Shrager & Siegler, 1998; Siegler & Jenkins, 1989). Finally, children and adults who frequently use decomposition and retrieval to solve arithmetic problems tend to have higher math performance and overall math achievement scores than those who depend on counting strategies (Carr & Alexeev, 2011; Carr et al., 2008; Fennema et al., 1998; Geary, Hoard, Byrd-Craven, & DeSoto, 2004).

2. Girls' arithmetic strategy use

The negative association between counting strategies and mathematics achievement suggests that a persistent preference for using counting strategies over mental strategies may be problematic. Girls, in particular, seem to be at risk for this problem. At every grade between kindergarten and third grade, girls are more likely than boys to solve problems using counting strategies. In contrast, boys are more likely than girls to solve problems mentally—using decomposition or retrieval (Carr & Davis, 2001; Carr & Jessup, 1997; Carr et al., 2008; Fennema et al., 1998; Jordan, Kaplan, Ramineni, & Locuniak, 2008). Girls' preference for counting strategies persists through primary school and on more complex math operations. They also abandon the use of concrete materials for counting out the answers to arithmetic problems more slowly than boys (Carr & Alexeev, 2011). At least through fifth grade, boys continue to use retrieval strategies more frequently than girls on arithmetic problems (Imbo & Vandierenendonck, 2007). In sixth grade, boys are more likely than girls to solve complex division problems mentally than with written algorithms (Hickendorff, van Putten, Verhelst, & Heiser, 2010).

This persistent preference for counting strategies may lead girls to have fewer opportunities to practice decomposition and retrieval, resulting in poorer accuracy when executing these mental strategies. In fact, across multiple ages, boys have been found to be more accurate than girls at using mental strategies when the task does not allow strategy choice and instead requires the use of mental calculation. Carr and Davis (2001) found that by first grade, boys were already more accurate than girls at using retrieval. Rosselli, Ardila, Matute, and Inozemtseva (2009) found that at 7-to-10 years of age, and again at 13-to-16 years, boys were more accurate than girls when required to solve addition, subtraction, multiplication, division, and fraction problems mentally.

The girls who demonstrate an early preference for mental strategies similar to that of boys, however, have later mathematics performance that is equal to that of boys. For example, Fennema et al. (1998) found that when tested on math performance in third grade, the subset of girls who had previously chosen to use invented strategies (such as decomposition) in second grade, performed just as well as the boys who had used invented strategies in the previous grade. These findings suggest that it is important to investigate individual differences within young girls relating to differences in their strategy choices.

In the literature on disadvantaged groups, within-group study has been important for uncovering factors that may help mitigate that disadvantage and to better understand how to promote the achievement of those who demonstrate cognitive disadvantage. In the present study, we have focused in depth on the different types of strategies young girls use to solve arithmetic problems in order to determine what cognitive factors predict for frequency of higher and lower level strategy use within girls.

3. Spatial and verbal processing and arithmetic performance

Two factors that potentially influence children's strategy choices are spatial and verbal skills. In fact, there is strong evidence from behavioral studies as well as from the field of neuroscience that both spatial and verbal processing are involved in generating the solutions to arithmetic problems (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Geary, Hamson, & Hoard, 2000; Kurdek & Sinclair, 2001; Lachance & Mazzocco, 2006; LeFevre et al., 2010; McLean & Hitch, 1999).

Object-based spatial skill is one type of spatial processing that has been found to relate to mathematics achievement in children. Object-based spatial measures include assessments of spatial visualization skills (such as the Block Design subtest from the WISC-IV; Coates & Lewis, 1984; Wechsler, 2003), and 2-d mental rotation tasks (such as the subtest of the Levine mental transformation task that requires children to match a picture of two halves of a shape rotated in 2-d space to four choices of possible completed figures; Levine, Huttenlocher, Taylor, & Langrock, 1999). (Note that in the present study, both these measures were used as components of the composite measure of spatial ability.) Recently, Levine and her associates (Gunderson, Ramirez, Bellock, & Levine, 2012) found that mental transformation ability predicted the quality of children's number line representations, and that number line representations mediated the relation between these spatial skills and later mathematics achievement. Other object-based spatial tasks shown to relate to children's math achievement include the ability to reproduce geometric designs (Geary & Burlingham-Dubree, 1989) and discriminating between similar shapes (Lachance & Mazzocco, 2006).

A second aspect of spatial processing that has been found to be related to mathematics achievement in children is spatial working memory. Spatial working memory refers to the capacity to maintain and simultaneously process visual–spatial information for short periods of time (Baddeley, 1992; Baddeley & Hitch, 1974). Studies have shown that children with mathematics disabilities perform worse on tasks measuring visual–spatial working memory than typically functioning control children (D'Amico & Guarnera, 2005; Mammarella, Lucangeli, & Cornoldi, 2010; McLean & Hitch, 1999).

Baddeley and Hitch (1974) proposed an influential multi-component model of working memory in which spatial and verbal information is processed through separate systems. Numerous studies have found evidence that these two systems function independently of one another (e.g., Brandimonte, Hitch, & Bishop, 1992). Moreover, neuro-psychological and neuroimaging studies have found distinct anatomical loci for the different working memory components (Henson, 2001; Vallar & Papagno, 2002).

In the research on children from preschool to adolescence, a large number of studies have examined the relation between spatial working memory and arithmetic performance relative to those of verbal working memory skills. These findings indicate that spatial and verbal processing contributes differently over the time course of acquiring arithmetic skills. Spatial working memory seems to be critical for the learning and application of new mathematical skills and concepts, whereas verbal working memory seems to be more important after a skill has been learned (LeFevre et al., 2010; Raghubar, Barnes, & Hecht, 2010). For example, spatial working memory has been found to be a unique predictor of first grade, but not second grade, mathematics achievement; whereas, verbal working memory has been found to be a unique predictor of second grade, but not first-grade, mathematics achievement (DeSmidt et al., 2009; McKenzie, Bull, & Gray, 2003). Similarly, LeFevre et al. (2010) found that in younger children, measures of spatial working memory predicted mathematical achievement independently of the linguistic or quantitative pathways. In summary, these different types of findings suggest that with young learners, spatial skills will have a stronger influence than verbal skills on choice of higher-level arithmetic strategies, as these students are still in the process of acquiring basic arithmetic knowledge.
4. Girls’ spatial skills

If spatial ability is important for executing mental strategies like decomposition and retrieval, then individuals with poorer spatial skills might choose to use these strategies less frequently. Girls as a group demonstrate poorer performance in the ability to mentally visualize and transform spatial information than boys. This gender difference in spatial processing has been found as early as three and four years of age (Levine et al., 1999; Vasileyva & Bowers, 2010), and extends across age levels. Gender differences in these types of spatial skills show the largest gender cognitive difference, close to one standard deviation (Linn & Petersen, 1985; Voyer, Voyer, & Bryden, 1995). Further, these differences in spatial skills have been found to mediate gender differences in the accuracy and speed of solving arithmetic problems in both children and adults (Geary, Saults, Liu, & Hoard, 2000; Rosselli et al., 2009).

Thus, we hypothesized that individual differences in spatial skills should be a particularly important factor in explaining variations in strategy use within young girls. It has been found that the relation between spatial skills and mathematics performance is stronger for girls than for boys (Casey, Nuttal, Pezaris, & Benbow, 1995; Friedman, 1995; Tartre, 1990), and the greater strength of these correlations for girls is found for arithmetic skills as well as other types of math abilities (Friedman, 1995). Thus, we examined whether girls with poorer spatial skills seem to have difficulty making the transition from counting strategies to higher-level mental strategies.

5. The present study

The present work differs from previous strategy research in its emphasis on understanding individual differences within young girls. We examined in depth each of the four main arithmetic strategies used by girls when attempting to solve arithmetic problems, as well as two cognitive processes which potentially contribute to these individual differences. First, we compared the frequency with which first grade girls apply the counting strategies (i.e., count-all and count-on) and the higher-level mental strategies (i.e., decomposition and retrieval) and then examined girls’ accuracy when attempting to use each of the strategies. We predicted that first grade girls would use count-all and count-on more frequently than decomposition and retrieval, based on previous studies of strategy preferences (e.g., Carr & Davis, 2001).

In terms of accuracy, individuals are believed to strategically select which strategy to use, based on which strategy would lead to the greatest accuracy given their current knowledge and the task demands (Shrager & Siegler, 1998). Girls may only choose to use decomposition and retrieval when there is a high likelihood that they will generate correct solutions. Thus, it was not clear which of the strategies would lead to the greatest accuracy.

Having established the pattern of strategy use among girls, the second and primary goal of this research was to compare the effects of verbal and spatial skills as possible underlying cognitive processes influencing young girls’ choice of arithmetic strategies. We proposed that the effects of spatial skills would be stronger than the effects of verbal skills on young girls’ choice of arithmetic strategies. Based on the literature, we predicted that greater spatial skills would be related to more frequent use of higher-level mental strategies, while lower spatial skills would be related to more frequent use of lower-level counting strategies.

6. Method

6.1. Participants

Participants were 127 first grade girls (mean age = 6.8 years, SD = 3.99) recruited from regular education classrooms in two school districts in the Boston area. Girls with individualized education plans for disabilities were excluded. Participants’ families lived in a wide range of socioeconomic conditions. The most common ethnic/racial groups represented in the sample were White (60.8%), Asian (14.7%), and Latino (9.8%).

6.2. Measures and procedures

6.2.1. Spatial skills

For each individual, an overall measure of spatial ability, a spatial composite score, was computed from the individual’s scores on three spatial tasks: the Block Design subtest of the WISC-IV, a 2-d mental rotation task, and a 3-d mental rotation task. Each of these tasks required children to mentally visualize, manipulate, and rotate objects in space, and to combine parts to make wholes. These measures (described in more detail below) were age appropriate for first graders and had been found to relate to math performance (Coates & Lewis, 1984; Levine et al., 1999). The spatial composite score for each individual was calculated by summing the z-score on the three spatial tasks.

6.2.1.1. WISC-IV Block Design subtest. The Block Design subtest of the Wechsler Intelligence Scale for Children—Fourth Edition (WISC-IV) (Wechsler, 2003) is designed for 6- to 12-year-olds and, is one of the most frequently used instruments for assessing children’s spatial skills (Caldera et al., 1999; Connor & Serbin, 1977, Serbin & Connor, 1979). Children are shown a series of 14 pictures of red and white 2-d patterns and asked to match each picture pattern by rearranging a set of small cubes so that the assembled top of the blocks matches the picture. Testing is terminated when a child fails three items in a row. Test–retest reliability and validity have been repeatedly established in socioeconomically diverse samples (Novak, Tshushima, & Tshushima, 1991; Wechsler, 2003). The full scale has an internal reliability of .97 and the Block Design subtest’s reliability is .86. This measure also has high test–retest reliability; with a mean test–retest interval of 32 days, the full scale has a stability reliability of .93 and the Block Design subtest’s reliability is .82 (Williams, Weiss, & Rolphus, 2003).

6.2.1.2. Two-dimensional mental rotation task. We also assessed girls’ spatial skills using a subset of 16 items from the 32-item Mental Transformation Task (Levine et al., 1999). These items, developed for 4- to 6-year-olds, require children to match a target—a picture of two halves of a shape rotated in 2-d space—to four choices of possible completed figures, in which only one of the four figures was actually formed by the two rotated halves. All of the items administered required mental rotation of the shape halves in order to match the whole figure. Specifically, children had to identify the correct whole shape from among four choices in a 2 × 2 array that could be formed from the halves.

There were two types of mental rotation problems (8 of each type presented in random order), which varied with respect to the relative positioning of the two target pieces. In the first type—the horizontal placement—each target piece was rotated 60° from the vertical axis, one clockwise and the other counterclockwise. In the second type—the diagonal placement—the target pieces were again rotated 60° from the vertical, but, in addition, the closest points of the pieces were separated by about 2 cm along both the horizontal and vertical axes, which placed the pieces diagonally from one another on the page (see Levine et al., 1999 for details on procedures and stimuli used for this task). The inter-item reliability of the items was α = .71 in the present study.

6.2.1.3. Three-dimensional mental rotation task. The third spatial task consisted of a 14-item 3-d mental rotation task that was adapted from a similar 10-item task developed for kindergarten age children. To make the task appropriate for first grade children, four additional items of increased difficulty were added. The task used 14 matching sets of two 3-d figures made out of multi-link cubes and covered in blue masking tape, as the stimuli. Children had to flip (3-d rotation) and turn (2-d rotation) one of the figures so that it was placed in the
same orientation as the other figure. The tasks increased in difficulty level across the 14 items by increasing the complexity of the figures in two ways—by adding more multi-link cubes to the core block structure and by increasing the number of directions that the cubes extended from this core. Participants were given 10 s to solve each problem. Within 10 s, the item was scored as correct if the first figure was in the same orientation as the second figure, or incorrect if the first figure was not in the same orientation as the second figure (see Casey et al., 2008 for detail on procedures and stimuli for this task). This mental rotation measure appeared valid (e.g., \( r = .45 \) with WISC-IV Block Design), despite inter-item reliability that was just above the minimum criterion (\( \alpha = .54 \)).

6.2.2. Verbal skills

We included a commonly used measure of receptive vocabulary, the Peabody Picture Vocabulary Test (PPVT-IV), to allow us to examine the role of verbal ability on strategy choice (Dunn & Dunn, 2007). This verbal measure is frequently used to predict for mathematical skills in the early school years, in part, because it is believed to capture the ability to acquire vocabulary in the number system and to develop lexical mappings between labels and their meanings (e.g., “4” is equal to 2 + 2) (Jordan, Hanich, & Kaplan, 2003; Klein, Adi-Japha, & Hakak-Benirez, 2010; Lefevre et al., 2009).

The PPVT is a norm-referenced measure of receptive vocabulary, and is administered by presenting a stimulus word (spoken by tester) and asking children to identify a pictorial representation of that word from four choices. This measure has demonstrated excellent reliability and validity in socioeconomically diverse samples (Dunn & Dunn, 2007), and inter-item reliability was high (\( \alpha = .90 \)) in the present sample.

6.2.3. Arithmetic performance

Girls solved 10 addition problems and then 10 subtraction problems, with problem order counterbalanced across individuals. Problems included either two 1-digit numbers, or one 2-digit number and one 1-digit number (adapted from Carr & Jessup, 1997). The problems used integers between 2–9 and 12–19, with the constraint that the same two integers (e.g., 3 + 3 or 6 – 6) were never used in the same problem. The range of sums was 9–24; the range of differences was 3–15.

Before beginning, the researcher placed color tiles on the table and told the child she could solve the problem in any way she chose (i.e., “You can count on your fingers, count on the counters, do the math in your head, or you might just know the answer”). Then, each problem was presented, written horizontally on a card (e.g., 7 + 3) and read aloud (e.g., “What is seven plus three?”). Children’s solutions and explanations were videotaped. One point was given for each item solved correctly. Both the set of addition and subtraction problems demonstrated excellent inter-item reliability (\( \alpha = .84 \) for addition; \( \alpha = .85 \) for subtraction).

6.2.4. Arithmetic strategies

Videos of girls completing the arithmetic tasks were used to code the strategy used on each problem. Two independent coders, who were research assistants on the project, assigned one of five possible codes to each of the 20 problems: count-all, count-on, decomposition, retrieval, and guess. Using Cohen’s kappa coefficient, we found that inter-rater reliability was .87, indicating close agreement between raters.

For each of the 20 problems, children’s overt behavior while solving the problem (e.g., counting aloud, using fingers) and verbal explanations were used to determine which strategy was used to solve the problem. When a child’s verbal explanation was inconsistent with behavior observed by the researcher (e.g., the child counted on her fingers, but reported “just knowing it”), the overt behavior was used to code the strategy. A strategy was categorized as retrieval if the child reported that they “just knew” the answer, and the speed of response was judged by the rater to be sufficiently fast to indicate memory retrieval. On a small number of problems (4%), the girls said “I don’t know” or reported guessing. These problems were coded as “guess” and were not used in any of the analyses.

7. Results

7.1. Average performance on measures of spatial and verbal skills

First, we examined average performance on the predictor variables (spatial and verbal skills). Examining each individual spatial task separately, the average standardized score on the Block Design subtest of the WISC-IV was 10.02 (SD = 3.01), the average mean proportion correct on the 2-d mental rotation task was .64 (SD = .20), and the average mean proportion correct on the 3-d mental rotation task was .61 (SD = .17). Individuals’ performance on the three spatial tasks was significantly correlated: For Block Design and 2-d mental rotation, \( r = .42 \), for Block Design and 3-d mental rotation, \( r = .45 \), and for 2-d mental rotation and 3-d mental rotation, \( r = .36 \). The average spatial composite score (− .04; SD = .78) for the three spatial measures was computed by taking the average of performance on each of the three spatial tasks converted to z-scores. We used the z-scores of the three spatial measures in a factor analysis, and found that they loaded on one factor, accounting for 61% of the variance, with the WISC measure loading .81, 3-d mental rotation measure loading .77, and the 2-d mental rotation measuring loading .75 on this factor. Thus, the discrete spatial tasks were weighted equally within the overall spatial composite measure. For the measure of verbal ability, girls’ average standardized score on the PPVT-IV was 107.53 (SD = 15.32).

7.2. Individual differences in frequency and accuracy of strategy use

7.2.1. Frequency of strategy use

To examine the frequency with which the girls used each of the four arithmetic strategies, we calculated the percentage of arithmetical problems on which individuals used each strategy. We then calculated the mean percentage of problems on which strategies were used across the sample of girls.

As shown in Table 1, the count-on strategy was used on the greatest percentage of problems (\( M = 44\% \), SD = 25.34), count-all second (\( M = 28\% \), SD = 27.69), decomposition third (\( M = 14\% \), SD = 18.85), and retrieval least (\( M = 11\% \), SD = 13.48). A Related-samples Friedman test comparing the distribution of the frequency of each category of strategy showed a significant difference, \( \chi^2 (3) = 92.54 \), \( p < .001 \). Further, all pairwise comparisons were significant. These results suggest that while first grade girls are able to use a variety of strategies, they are more likely to choose to use a counting strategy than the more sophisticated decomposition and retrieval mental strategies.

7.2.2. Accuracy of strategy use

Overall, girls were quite accurate at solving the arithmetic problems; an average of 80% (SD = 20.33) of problems were answered correctly. We conducted a Linear Mixed-Model analysis to determine whether differences in the mean accuracy on the problems were related to the strategy used to solve them. Using a Linear Mixed Model allowed us to account for the within-subject correlations between strategy choice and accuracy across problems by using the series of arithmetic problems as the repeated measures variable (Wallace & Green, 2002).

Strategy choice was found to be related to accuracy, \( F(4, 2488) = 79.05 \), \( p < .0005 \). As shown in the second column of Table 1, girls were most accurate when they used retrieval, next decomposition, and then count on. Girls were least accurate when they used the most basic, count-all strategy. Pairwise comparisons indicated that girls’ accuracy was significantly lower when they used the count-all strategy than when they used retrieval.
7.3. Analyses examining predictors of strategy choice

7.3.1. Correlations

First, we examined the relation among the individual spatial measures, the verbal measure, and the frequency of use of the different strategy types. As shown in Table 2, a similar pattern of significant relations was found for the three spatial measures when correlated with the frequency of use of the different types of arithmetic strategies. Consequently, in the series of regression analyses, we used the composite measure rather than the individual spatial skills measures.

7.3.2. Regression analyses

Standard linear regressions were conducted to determine how much variance in girls’ use of each of the four arithmetic strategies was accounted for by verbal and by spatial skills. For each of the four types of arithmetic strategies, verbal and spatial skills were entered simultaneously. Separate analyses is a commonly used procedure to examine factors predicting strategy use (e.g., Carr et al., 2008; Imbo, Vandierendonck, & Rosseel, 2007; Lemaire & Siegler, 1995); thus, we used this approach to make it possible to compare findings with the studies most frequently cited in the field. As shown in Table 3, across the four regression analyses, we found that verbal and spatial skills together accounted for 12% of the variance in the use of count-all, 28% in the use of decomposition strategy, and 13% in the use of retrieval. No significant variance was explained for the count-on strategy.

Specifically, we predicted that the effects of spatial skills would be stronger than the effects of verbal skills on young girls’ choice of arithmetic strategies. To test this, we examined the amount of variability in girls’ higher-level mental strategy choices (decomposition and retrieval) explained by spatial skills after controlling for variance explained by their verbal skills. In the next sections, we discuss the relative contributions of verbal and spatial skills on frequency of use of each strategy in each of the regression analyses. We first present the two regressions for the counting strategies and then present the regressions for the mental strategies. See Table 3 for the detailed descriptions of the four regression analyses.

7.3.2.1. Predictors of counting strategies. As illustrated in Table 3, when both verbal and spatial skills were simultaneously entered into the regression equation, it was spatial rather than verbal skills that negatively predicted frequency of use of count-all. A one-point decrease in girls’ spatial composite score is associated (on average) with an 11.63% decrease in the use of the count-all strategy. In contrast to count-all, for the problems in which the count-on strategy was chosen, there was no association between either spatial or verbal scores and the dependent measure.

7.3.2.2. Predictors of mental strategies. For the higher-level retrieval strategy, it was spatial and not verbal skills that significantly predicted the frequency with which young girls used the retrieval strategy. For decomposition, in contrast to retrieval, spatial as well as verbal skills significantly predicted the frequency with which girls used decomposition. The two regression analyses indicated positive partial effects such that a one-point increase in girls’ spatial composite score is associated (on average) with a 7.97% increase in the average use of decomposition and a 6.8% increase in the average use of retrieval.

7.3.2.3. Predictors of overall accuracy on the arithmetic problems. The findings on predictors of overall accuracy on the arithmetic tasks were consistent with the findings on predictors of frequency of use of the retrieval strategy (see Table 3): it was spatial, and not verbal skills, that significantly predicted how accurate the girls were on the arithmetic tasks. The Bs in the regression for the two regression analyses indicated positive partial effects such that a one-point increase in girls’ spatial composite score is associated with a 6.82% increase in average accuracy on the arithmetic task.

8. Discussion

8.1. Relations between frequency of strategies used and accuracy

In the literature, it has been found that during the primary years, girls, relative to boys, are more likely to depend on counting strategies (rather than mental strategies) to solve arithmetic problems (Carr & Davis, 2001; Carr & Jessup, 1997; Carr et al., 2008). Consistent with that literature, in the present study, while first grade girls were able to use a variety of strategies, they were more likely to choose to use a count-on or count-all counting strategy than the more sophisticated decomposition and retrieval mental strategies. The least efficient count-all strategy was used fairly often, but it was also the strategy least likely to yield an accurate solution; girls who chose to use this strategy were not as accurate as those who chose to use retrieval (see Table 1). It could be argued that use of this “more primitive” counting strategy is simply an effect of lower “general intelligence” in the girls preferring to use this strategy. However, this explanation is unlikely because only spatial skills (and not verbal skills) predicted for the frequency of using this strategy. Thus, rather than a deficit in general intelligence, it is likely to be a more specific spatial deficiency causing the girls to prefer to use this lower-level strategy.

Table 1

<table>
<thead>
<tr>
<th>Mean frequency of strategy use</th>
<th>Mean accuracy</th>
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<tbody>
<tr>
<td>% (SD)</td>
<td>% (SD)</td>
</tr>
<tr>
<td>Count-all</td>
<td>28% (27.69)</td>
</tr>
<tr>
<td>Count-on</td>
<td>44% (25.34)</td>
</tr>
<tr>
<td>Decomposition</td>
<td>14% (18.85)</td>
</tr>
<tr>
<td>Retrieval</td>
<td>11% (13.48)</td>
</tr>
<tr>
<td>Guessing</td>
<td>4% (8.58)</td>
</tr>
</tbody>
</table>

Note: Four percent of the trials were coded as guess and are not included in analyses.

Table 2

<table>
<thead>
<tr>
<th>Correlations between the frequency of the four strategy types and scores on the spatial and verbal measures.</th>
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<tbody>
<tr>
<td>WISC-IV Block Design</td>
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<tr>
<td>----------------------</td>
</tr>
<tr>
<td>Count-all</td>
</tr>
<tr>
<td>Count-on</td>
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<tr>
<td>Decomposition</td>
</tr>
<tr>
<td>Retrieval</td>
</tr>
</tbody>
</table>

**p < .05.
***p < .01.

Table 3

<table>
<thead>
<tr>
<th>Five linear regression models with verbal and spatial skills as predictors of the frequency of the four types of strategies used and the overall arithmetic accuracy.</th>
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<tbody>
<tr>
<td>Dependent measures for each regression analysis</td>
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<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Counting strategies</td>
</tr>
<tr>
<td>Spatial</td>
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<tr>
<td>Count-on</td>
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<tr>
<td>Spatial</td>
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<td>Mental strategies</td>
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<tr>
<td>Verbal</td>
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<tr>
<td>Retrieval</td>
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<tr>
<td>Spatial</td>
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</tbody>
</table>

*p < .05.
**p < .01.
Counting has been found to be a back-up strategy, with children going back to concrete fingers or counters when the arithmetic problem becomes too hard for them to do mentally; children often revert to this approach when they are trying to solve more difficult problems (Ashcraft & Fierman, 1982; Siegler & Jenkins, 1989). However, using fingers or counters on difficult problems that involve large numbers may lead to errors in counting or keeping track of what has been counted. Nevertheless, it should be noted that the young girls in this study on average were high performers across all four types of strategies, with accuracy when using count-all close to 80% and accuracy on retrieval over 90% correct.

Siegler and his associates (e.g., Shrager & Siegler, 1998) have found that in terms of accuracy, individuals strategically select which strategy to use, based on which strategy would lead to the greatest accuracy given their current knowledge and the task demands. Girls may only choose to use decomposition and retrieval when there is a high likelihood that they will generate correct solutions. Thus, it may be that the frequency with which young girls attempt to use higher-level strategies, rather than their accuracy using particular strategies, is most indicative of their present and future mathematics competence. For example, it has been found that young girls who demonstrate an early preference for mental strategies similar to that of boys have later mathematics performance that is equal to that of boys (Fennema et al., 1998).

This preference for counting strategies, therefore, may have long-term implications for girls’ mathematics achievement. The use of counting strategies is negatively associated with later mathematics achievement (Carr & Alexeev, 2011; Carr et al., 2008; Fennema et al., 1998; Geary et al., 2004). Second graders who solve arithmetic problems using the mental strategies more frequently than counting strategies have higher mathematics achievement scores in fourth grade than those second graders who used counting strategies more often (Carr & Alexeev, 2011).

8.2. Spatial and verbal abilities as predictors of arithmetic strategy choice and arithmetic accuracy

The present results make a contribution to the literature by demonstrating that early object-based spatial skills influence not only the outcome of arithmetic operations in students (e.g., Gunderson et al., 2012), but also the choice of strategies used to solve arithmetic problems. We found that spatial skills assessing spatial visualization (as measured by the WISC-IV Block Design subtest), and mental rotation ability (as measured with 2-d and 3-d tasks) positively predicted frequency of use of both retrieval and decomposition in young girls. In addition, consistent with this pattern, these types of spatial skills negatively predicted frequency of use of the least sophisticated strategy, count all. Only for decomposition, did verbal skills, along with spatial skills, contribute to the frequency of use. Nevertheless, these findings with first grade girls are consistent with the hypothesis that spatial skills contribute to the use of higher-level mental strategies at the outset of schooling. It is also important to point out that spatial skills (and not verbal skills) predicted for overall accuracy of performance as well as higher level strategy choice.

It should be noted that spatial skills only explained a relatively small proportion of variance in the frequency of choosing higher-level mental strategies (9% to 13%), with total variance explained in the regressions higher for decomposition (28%) than for retrieval (13%). The measure of 3-d mental rotation used in this study was not as reliable as the other measures of spatial skills assessed in the study. Future research would benefit from considering other ways of assessing spatial skills in young children to better understand the association between early spatial skill and individual differences in arithmetic strategies.

It is also quite possible that other factors not analyzed in the current study have a role in predicting frequency of use of the different types of strategies that the girls used. In fact, several factors, including problem difficulty and individual differences in working memory, arithmetic skill and confidence levels have been found to be related to strategy choice across genders (e.g., Ashcraft & Fierman, 1982; Geary et al., 2004; Siegler, 1988). Further, it should be noted that the present study assessed specific kinds of object-based spatial skills—those relating to spatial visualization and mental rotation ability in young learners. In future research, it would be useful to determine whether this relation holds for other types of spatial visualization skills such as the Embedded Figure Test (Oltman, Raskin, & Witkin, 1971), or is found with measures of spatial perception ability such as the Water Level task (Vasta & Liben, 1996).

Nevertheless, the pattern of predictive relations found across different strategies in the present study is revealing. It is noteworthy that young girls with better object-based spatial skills approach arithmetic in ways that mirror those of young boys (who tend to have better spatial skills). It is consistent with previous findings indicating the importance of spatial skills both in accounting for individual differences within girls’ math achievement, and mediating gender differences in mathematics later in development (Casey, Nuttall, & Pezaris, 1997; Casey et al., 1995; Friedman, 1995; Geary, Saults, et al., 2000; Tartre, 1990).

8.2.1. Predictors of retrieval

Spatial, and not verbal skills, were found to be predictive of the frequency with which girls used retrieval. Strikingly, almost no variance in first grade girls’ use of retrieval was explained by their verbal skills. This result is in contrast to studies of adults, which have found language processing to be an important predictor of math-fact retrieval (e.g., Grabner, Ansari, Koschutnig, Reishofer, Ebner, & Neuper, 2009). The findings are consistent, however, with more recent findings that suggest that younger arithmetic learners may depend more on spatial than verbal processing when using retrieval (Cho, Ryali, Geary, & Menon, 2011). In this study, girls were only asked to solve simple arithmetic problems (i.e., sums less than 24 and differences less than 15); thus, it is possible verbal abilities may be involved in girls’ choice of strategies for solving more complex arithmetic problems.

It may be that early in development, when arithmetic facts are not yet highly practiced, retrieval relies on associating the relative spatial position of numbers on a mental number line, rather than a pure semantic association of problem and response. Girls with good spatial skills may have the advantage of being able to represent numerical magnitude spatially, which in turn, may enable them to use higher-level strategies when solving arithmetic problems. In fact, Levine and her associates (Gunderson et al., 2012) recently found that number line representation ability assessed at age six mediated the relation between spatial skills assessed at age five and arithmetic performance assessed at eight years of age.

8.2.2. Predictors of decomposition

In contrast to retrieval, the frequency with which girls used decomposition (another higher-level mental strategy) was predicted by verbal skills as well as spatial skills. Both decomposition and retrieval require recall of previous math facts. There are, however, substantial differences between the mental procedures involved in applying these two types of mental strategies. Decomposition, but not retrieval, involves the use of a series of analytical steps, such as subtracting from one of the addends and then adding the value subtracted back. For example, to use decomposition to solve 6 + 7, a child must subtract from one of the addends to reduce the problem to a simpler, known problem (e.g., subtract 1 from 7 to get 6 + 6), retrieve the answer to the simpler problem and maintain it in working memory while simultaneously remembering the value that was subtracted, and, finally, add the value subtracted to the sum that was retrieved (e.g., 12 + 1 = 13). We propose that these additional steps may depend on verbal/analytical reasoning and verbal working memory, which may help to explain why decomposition draws on both verbal and spatial skills. An interesting direction for future research might be to examine, in depth, the different underlying spatial and verbal long-term and working memory requirements of retrieval and decomposition strategies.
8.2.3. Predictors of counting strategies

The present results show that the girls who generally choose to by-pass mental calculations by depending most heavily on count-all, a basic counting strategy, are likely to have lower spatial skills. These girls, with the poorest spatial skills, may have difficulty mentally manipulating numbers in their heads, and thus revert most frequently to the back-up strategy of counting all the numbers concretely.

Interestingly, for the problems in which a count-on strategy was used, there was no significant association between the frequency of choosing this type of strategy and either verbal or spatial skills. This strategy was common among almost all the girls, and in fact was the strategy used most frequently by them. The prevalence of this strategy might reflect the fact that it is often directly taught to children in kindergarten and first grade programs. Thus, the use of this strategy may depend less on individual differences in cognitive factors and more on the differences in instructional practices. Another possible explanation is that the use of the count-on strategy, in particular, might be related to other spatial skills not measured in this study. Last, there are individual differences in whether the count-on strategy is applied using concrete manipulatives or whether it is achieved through counting aloud or counting mentally. This difference in approach may be critical—as the importance of particular underlying cognitive processes may depend in part on how the count-on strategy is executed. Thus, it may be useful in future research to include measures of other spatial skills and to further distinguish between girls who are executing this strategy by counting using fingers or counters, and those who are able to apply it by counting aloud or in their heads, without having to depend on concrete manipulatives.

9. Conclusions

The major purpose of the present research was to compare verbal and spatial skills as cognitive factors predicting the frequency of using the higher-level mental strategies of decomposition and retrieval in young girls. Although the pattern of relations showed some difference between the two types of mental strategies, across both types, we found that at the outset of acquiring arithmetic skills, young girls who have poor object-based spatial skills (relating to spatial visualization and mental rotation) are at a greater disadvantage in terms of their strategy choice (they are less likely to use higher-level mental strategies for solving arithmetic problems) than girls with more effective spatial skills. Verbal skills only explained additional variance in the frequency of use of decomposition—a higher level and more complex strategy—that appears to draw on both verbal and spatial skills.

Our focus on girls does not allow us to speak to whether the evident patterns of association are unique to girls, or whether they may, in fact, generalize equally well to boys. In short, without dismissing the additional value of future studies designed to answer the question of whether our findings generalize to boys, we believe there remains considerable value in first understanding the mechanisms that predict higher or lower arithmetic and spatial skills among girls, who presently (as a group) demonstrate relative disadvantage in these domains.

Our study showed the influence of early spatial skills on young girls’ arithmetic strategies. To directly determine the educational implications of these findings, an important research step would be to determine whether early spatial training can increase the frequency with which girls are able to use higher-level strategies and decrease their dependence on the most basic counting strategy. It has been found in previous research that spatial training can substantially improve spatial skills, particularly in the early grades (Hand, Uttal, Marulis, & Newcombe, 2008). Thus, one means of improving girls’ mathematics trajectories in general might be to start early by placing a greater emphasis on developing spatial reasoning skills in the early primary mathematics curriculum (e.g., Casey, Erkut, Ceder, & Young, 2008).

References
